Comparison of Model Reference Adaptive Control and Cascade PID Control for ASTank2

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Abstract—This paper presents Model Reference Adaptive Control (MRAC) compared to Cascade PID control in order to evaluate their performances. The cascade control has an outer loop for the level control and an inner loop for flow control that can make the response faster. There the linearization technique is applied to obtain a new linear model. MRAC is designed based on Lyapunov theory and Barbalet’s lemma that will make the system stable and obtain convergences. The reference model can be selected as the third order system with relative degree two to satisfy rise time, settling time, peak time, and overshoot requirements. General adaptive control laws are developed for tuning three gains based on the reference model. These control gains cover level tracking performance. MRAC also help to remove dynamic uncertainties and modeling errors. Simulation results on Matlab/Simulink indicate that the MRAC can improve the control performances compared to those of the classical PID control effectively.

Keywords-component: MRAC, Coupled tank, Nonlinear model, Process control, Cascade control, Adaptive control.

I. INTRODUCTION

Process control design of tanks using nonlinear techniques is attracting researchers. Jiffy Anna John et al [1] introduced a level backstepping control strategy for nonlinear coupled tanks system. The objective of the control algorithm is to track the desired level of liquid in the second tank by using flow rate of liquid into the first tank as the manipulated variable. Similarly Vasile Calofir et al [2] presented a level backstepping control system for another coupled tank process. The backstepping method is also proposed to control flow and temperature process of a tank system [3]. The designed control system is asymptotically stable for the plant without uncertainties and globally uniformly bounded for the plant with uncertainties. In addition, the backstepping control is compared to the feedback linearization to evaluate their performances [4]. In the special case, Haizhou Pan et al [5] have done experiments of a nonlinear backstepping liquid level controller for a state coupled two tank system. The experiments indicated that with backstepping control, the level feedback could track well those of simulation. However, there exist some vibrations and static errors due to noises of sensors and modeling errors. Another continuous stirred tank reactor also uses the backstepping control technique to obtain a stable control system [6]. There the performance of the backstepping control is compared with that of the standard PI controller. Recently, Houssemédine Gouta et al [7] also present a comparative study between model-based predictive and backstepping controllers for a state coupled four tank system with bounded control inputs. Simulation and experiments indicate that the backstepping control method is easier to design and implement but the tracking error for the predictive controller is better.

Another interesting method that helps to construct stable control systems is Model reference adaptive control (MRAC). M. Saad et al [8] provide the performance comparison between PI and MRAC for coupled-tank system. The simulation results on Matlab/Simulink demonstrate that the MRAC controller produces better responses and performances. Another MRAC [9] also shows good performances.

Although the backstepping control methods work well for the nonlinear process of tank systems, it cannot eliminate perfectly dynamic modeling errors and uncertainties. Moreover, Backstepping algorithms are very difficult to realize on PLC controllers. Therefore in this paper, MRAC is proposed and simulated on Matlab/Simulinks for the coupled tanks (ASTank2) as an extension of previous works [10-11]. MRAC is proposed to achieve better force tracking performances as well as obtain convergences. The reference model is selected as the third order relative two to satisfy requirements of rise time, settling time, peak time, and overshoot of the force tracking. Simulation results will provide comparison between this proposed control algorithm and cascade PID control that uses popularly in the process automation.

II. SYSTEM DESCRIPTION

A process configuration diagram of ASTank2 is shown in Fig. 1. The system consists of two water tanks placed at the same height. The tanks can be used as...
filling/drainage process independently or coupled by opening tap RC. Water drainage from the tanks is achieved by adjusting taps R11 and R12. Water is pumped in the process using the main pump (P) or the auxiliary pumps (P1, P2). The main pump is inverter-driven while the auxiliary pumps are constant flow and have been designed to work as external disturbances for the level control. The water level in the tanks can be measured by the pressure transducers fitted on the bottom, named as LT1 and LT2. The water flow in the two tanks can be controlled independently by using the proportional valves FC1 and FC2 while the volumetric flow rates are measured by the transducers FT1 and FT2. Two static pressure sensors (PT1, PT2) have been fitted on the feed line between the pump drainage and the T junction that branches into the two tanks. The right tank is rectangular with an inner cross section of 150mm x 140mm and a height of 500mm. The left tank has the upper part same cross section as the right tank and then continues with a sloped wall at an angle of 75deg. The bottom cross section is rectangular with a width of 140mm and a length of 80mm. The height of the rectangular part is 250 and total height amounts to 500mm.

III. CASCADE PID CONTROL

The system model should be obtained to do process simulation and control design. The modeling can be started from the first principle conservation of volume equation and achieved below:

\[
A_1 \frac{dh_1}{dt} = q_{in1}(t) - a_1 \sqrt{2gh_1(t)} + \text{sgn}(h_2(t) - h_1(t))a_1 \sqrt{2gh_2(t) - h_1(t)}
\]

\[
A_2 \frac{dh_2}{dt} = q_{in2}(t) - a_2 \sqrt{2gh_2(t)} - \text{sgn}(h_2(t) - h_1(t))a_2 \sqrt{2gh_2(t) - h_1(t)}
\]

where,

- The cross sectional area of Tank 1, \( A_1 = 0.021 \text{m}^2 \) if \( h_1 \geq 0.025 \text{m} \), \( A_1 = 0.0112 + 0.0385h_1 \text{m}^2 \) if \( h_1 < 0.025 \text{m} \); the cross sectional area of Tank 2, \( A_2 = 0.021 \text{m}^2 \).
- The cross sectional areas of outlet pipe in the Tank 1 and Tank2 are \( a_1 = 0.000023 \text{ m}^2 \) and \( a_2 = 0.000023 \text{ m}^2 \), respectively. These values can be obtained by experiments.
- The cross sectional area of interaction pipe between the Tank 1 and Tank2 are \( a_c = 0.000023 \text{ m}^2 \). This value also can be found from experiments.
- The acceleration due to gravity is \( g = 9.81 \text{m/s} \).
\[ q_{in1}(t) = A_1(h_1) \left( -c_1 h_1(t) + u_1(t) + \frac{a_1}{A_1(h_1)} \sqrt{2gh_1(t)} \right) - \frac{a_1}{A_1(h_1)} \text{sgn}(h_2(t) - h_1(t)) \sqrt{2gh_2(t) - h_1(t)} \]
\[ q_{in2}(t) = A_2(-c_2 h_2(t) + u_2(t) + \frac{a_2}{A_2} \sqrt{2gh_2(t)} + \frac{a_c}{A_2} \text{sgn}(h_2(t) - h_1(t)) \sqrt{2gh_2(t) - h_1(t)} \]

where \( c_1, c_2 \) are constants for the linearization of dynamic model and \( u_1, u_2 \) are the virtual control inputs which are outputs of PID controller.

Therefore the nonlinear dynamic system is rewritten as two linear independent models:
\[ \dot{h}_1(t) = -c_1 h_1(t) + u_1(t), \quad \dot{h}_2(t) = -c_2 h_2(t) + u_2(t) \]  

The cascade PID control diagram has two loops. The inner loop is flow control and the outer loop is the level control. The virtual control inputs \( u_1, u_2 \) must satisfy the upper and lower limitations depended on the maximum and minimum values of \( q_{in1}(t) \) and \( q_{in2}(t) \).

**IV. MODEL REFERENCE ADAPTIVE CONTROL**

A structure of model reference adaptive control for the right or left tank is constructed as shown in Fig. 4. There the linear tank model for designing controllers is chosen as in Equation (5). The flow control loop including the valve model and controller is selected as 
\[ H_v = \frac{K}{s+b} \]; the linearization function \( Q \) is presented in (3) for the tank 1 and (4) for the tank 2. The model reference adaptive controller is designed with three parameters as a feed forward gain \((K1)\), feedback gain \((K2)\) and filter gain \((K3)\) to improve tracking performances and reduce effect of noises from sensors. The closed loop transfer function is derived as:
\[ H = \frac{h}{h_d} = \frac{KK_0Q(s+a)}{(s+a)(s+b)(s+c)+KK_0Q(s+a)} \]

where \( h_d \), \( h \) are the desired and feedback values, respectively.

Therefore, the reference model can be selected as the third order system with relative degree two as follow:
\[ H_m = \frac{K_m(s+z_1)}{(s+p_1)(s^2+2\xi\omega_ns+\omega_n^2)} \]

where \( \xi \) is damping ratio, \( \omega_n \) is natural frequency, \( p_1 \) is a real pole, \( z_1 \) is zeros.

The damping ratio and natural frequency determines locations of two complex poles. The damping ratio can be increased to reduce overshoot while the natural frequency is used to adjust settling time. The rise time also can be reduced when the negative real pole \(-p_1\) is selected far from the image axis. The zeros \( z_1 \) should be selected as negative real values so that the minimum phase is satisfied. The overshoot is obtained by choosing proper zeros. By choosing parameters \( \xi, \omega_n, p_1, z_1 \) the reference model \( H_m \) can be obtained to satisfy the requirements of overshoot, settling time, rise time and peak time. In addition the model \( H_m \) should satisfy requirements of strictly positive real transfer function (the real part of \( H_m \) is positive for every frequency) so that the stability of MRAC will be satisfied.

The control law of MRAC is calculated as:
\[ U = K_1U_1 + K_2U_2 + K_3U_3 \]
where $U_1 = h_d$, $U_2 = -h$, $U_3 = -\frac{U}{s + a}$. This control law also can be considered in the time-domain as:

$$u(t) = \sum_{i=1}^{3} K_i(t) u_i(t) = K_1(t) u_1(t) + K_2(t) u_2(t) + K_3(t) u_3(t)$$

(9)

If all parameters of real system are known and the modeling is fully perfect, the ideal control gain $K^* = [K_1^* \ K_2^* \ K_3^*]^{T}$ is identical as $K = [K_1 \ K_2 \ K_3]^{T}$. However the modeling has the small error and some dynamic parameters are unknown so the ideal control gain $K^*$ cannot be found. The ideal control law is calculated as:

$$u^* = \sum_{i=1}^{3} K_i^*(t) u_i(t) = K_1^*(t) u_1(t) + K_2^*(t) u_2(t) + K_3^*(t) u_3(t)$$

(10)

The output of system equals the output of reference model for the ideal control law as $h = h_m = H_m U_1 = H_m h_d$.

Because the ideal control gains are unknown so estimated control gains $\hat{K} = [\hat{K}_1 \ \hat{K}_2 \ \hat{K}_3]^{T}$ are defined so that the output of system can track that of reference model. The errors between ideal control gains and estimated control gains are defined as:

$$\Delta \hat{K} = \hat{K} - K^*$$

(11)

So the estimated control gains can be recalculated as:

$$\hat{K} = K^* + \Delta \hat{K}$$

(12)

The estimated control law $\hat{u}$ is also calculated as:

$$\hat{u} = \hat{K}^T U = K^* U + \Delta K^T U$$

$$= K_1^* \left( U_1 + \frac{\Delta K^T U}{K_1^*} \right) + \sum_{i=2}^{3} K_i^* U_i$$

(13)

If define $\hat{U}_i = U_i + \frac{\Delta K^T U}{K_1^*}$, the estimated control law is then obtained:

$$\hat{u} = K_1^* \hat{U}_1 + \sum_{i=2}^{3} K_i^* U_i$$

(14)

This estimated control law has the same form as (10). It contains ideal control gains and $\hat{U}_i$ can be considered as $U_i$ so;

$$h = H_m \hat{U}_1 = H_m \left( U_1 + \frac{\Delta K^T U}{K_1^*} \right)$$

(15)

The error between outputs of system and reference model is;

$$e = h - H_m U_1 = H_m \left( \frac{\Delta K^T U}{K_1^*} \right)$$

(16)

The Lemma 8.1 [12] indicates that if $H_m$ is strictly positive real transfer function, the adaptive law can be selected as;

$$\Delta \hat{K} = -\text{sign}(K_1^*) \gamma e U$$

(17)

where $\gamma$ are given positive constants.

Because the ideal control gain $K_1^*$ is positive, the control law is obtained as;

$$\Delta \hat{K} = -\gamma e U$$

(18)

The adaptive law $\Delta \hat{K} = \frac{\hat{K} - K^*}{\gamma e U}$, lead to

$$\hat{K} = K^* + \gamma e U$$

(19)

or

$$\hat{K}_1 = -\gamma_1 e U_1, \ \hat{K}_2 = -\gamma_2 e U_2, \ \hat{K}_3 = -\gamma_3 e U_3$$

(20)

Model reference adaptive control for ASTank2 is constructed on Matlab/Simulink as shown in Fig. 5. The reference model is selected as a stable system and its output will converge to the desired level $h_d$. Therefore it is only necessary to consider the stability and convergence of $e = H_m \left( \frac{\Delta K^T U}{K_1^*} \right)$. The reference model $H_m$ is chosen as the third order system with relative degree two, and substituted to the error equation, leads to:
\[ e = \frac{b_1s + b_2}{b_3s^3 + b_4s^2 + b_5s + b_6} \ u_e \]  

(21) 

where \( u_e = \frac{\Delta K^T U}{K_1^*} \). The error equation can be rewritten in the state space form as:

\[ \dot{X} = AX + Bu_e \]

\[ e = C^T X \]  

(22)

The reference model \( H_m \) can be selected to satisfy the requirements of strictly positive real transfer function (check on MATLAB). The Kalman-Yakubovich lemma indicates that there exists symmetric positive matrix \( P \) and \( Q \) so that the following equation is satisfied.

\[ C P B = Q \]

(23)

The Lyapunov function of \( X, \Delta K \) is selected as:

\[ V = X^T P X + \frac{1}{\gamma K_1^*} \Delta K^T \Delta K \]  

(24)

Taking its derivative to obtain \( \dot{V} \leq 0 \) so the system is stable and \( X, \Delta K, e = C^T X \) are bounded. If \( U \) is bounded then \( X \) is bounded and \( \dot{V} = -X^T QX \) is also
bounded. The conditions of Barbalet’s lemma are satisfied as:

1. $V$ is lower bounded
2. $\dot{V}$ is negative semi-definite
3. $\dot{V}$ is uniformly continuous (because $\ddot{V}$ is bounded)

This means $\dot{V}$ goes to zeros when time goes to infinity. So $e = C^T X$ converges to zeros.

V. SIMULATION RESULTS

The simulation program on Matlab/Simulink is constructed as shown in Fig. 5. The reference models are chosen as (25) and (26).

$$H_{m1} = \frac{(s + 0.12)}{(s + 2)(s^2 + s + 0.06)}$$

$$H_{m2} = \frac{(s + 0.03)}{(s + 0.5)(s^2 + s + 0.06)}$$

The parameters of linearization models are selected bases on the time delay of real system as $c_1 = 0.05$, $c_2 = 0.05$. The PID parameters for two tanks are $K_p = 0.25$, $T_1 = 10$, $T_d = 0$. The convergence speeds of MRAC are $\gamma_1 = \gamma_2 = \gamma_3 = 1.2$. The simulation results for the cascade PID control are compared with those of MRAC in Fig. 6 and Fig. 7 indicate that MRAC produce the outputs with no overshoot and smooth level responses. Whereas the cascade PID control has the overshoots of 10%. Both MRAC and cascade PID control have the same settling time of 110 sec. Moreover, the flow rate responses of these control systems can satisfy the maximum limitation of 0.0001 m$^3$/s.

VI. CONCLUSION

In this paper two type of control schemes for the ASTank2 have been presented. The first one is the cascade PID control that has proven popularity in process automation and the second one is the Model reference adaptive control that has shown lots of advantages. This proposed adaptive control will not only provide a stable control system but also eliminate modeling errors and dynamic influences in order to obtain good performances. The simulation results on Matlab/Simulink indicate that the MRAC outputs the response with no overshoots compared to those of cascade PID control.

In the future the experiments will be done to have comparison with simulation results.

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REFERENCES


